Effects of decoherence on the shot noise in carbon nanotubes

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We study the zero-frequency noise in an interacting quantum wire connected to leads, in the presence of an impurity. In the absence of quasiparticle decoherence the zero-frequency noise is that of a noninteracting wire. However, if the collective, fractionally charged modes have a finite lifetime, we find that the zero-frequency noise may still exhibit signatures of charge fractionalization, such as a small but detectable reduction in the ratio between the noise and the backscattered current (Fano factor). We argue that this small reduction in the Fano factor is consistent with recent observations of a large reduction in the experimentally inferred Fano factor in nanotubes (calculated assuming that the backscattered current is the difference between the ideal current in a multiple-channel noninteracting wire and the measured current).

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Shot noise has been long used to extract information about the nature of the elementary excitations in a system. Thus, shot-noise measurements have been used extensively to study exotic phases, such as the ones arising in strongly interacting one-dimensional systems. For example, they confirmed the presence of fractionally charged quasiparticles in a fractional quantum Hall fluid.[1] While various proposals have been made to use shot noise to characterize other one-dimensional systems such as carbon nanotubes,[2–6] a clear confirmation of the fractionalized nature of the elementary quasiparticles in nanotubes is still lacking. The major effect that obscures the fractional character of the excitations in nanotubes (and other nonchiral one-dimensional systems) is the presence of metallic leads: at small frequencies the noise probes the physics of the noninteracting leads and not the physics of the nanotube. Thus, while in a chiral Luttinger liquid (LL) (such as a fractional quantum Hall effect edge state system), the Fano factor, defined as the ratio between the noise and the backscattered current, is equal to the fractional charge ge, in a nanotube the Fano factor has been predicted[3] to be simply equal to the electron charge e.

Recent experiments[7,8] have measured the Fano factor in nanotubes and observed nevertheless a reduced Fano factor. The value of the observed Fano factor in Ref. 7 is roughly $0.4e$—too small to be explained by the noninteracting physics of the leads[3] but also too large to be consistent with the presumed value of the fractional charge in nanotubes $(0.2e)$.[2]

We propose that this reduction in the Fano factor is the result of a combination of three factors: the first are the interactions in the wire. The second is the quasiparticle decoherence. We conjecture that, in connection with the electronic interactions, this will yield the modification of the ideal conductance of a nanotube in contact with leads from $4e^2/h$ to $4g_e e^2/h$, even in the absence of impurities (g_e is an effective "fractional-charge"-type parameter that is close but not equal to 1). We justify this conjecture by using the Luttinger liquid formalism and by introducing a finite lifetime for the fractionally charged quasiparticles. The physical factors that may have a role in the appearance of such a lifetime are the (usually neglected) band curvature at higher energies, various inelastic processes, or other types of interactions between electrons (three-body interactions, etc.). For example, the effects of band curvature were treated in a quite a few

works over the past years[9,10] and indeed the Green's functions, the self-energy, and properties such as the spectral function were found to be affected by the band curvature. In the future it would be interesting to calculate in this framework the quasiparticle lifetime as well as the conductance of a finite-size wire in contact with metallic leads. This is however an involved project, which is beyond the scope of the present paper and will be addressed in a separate work.

The third factor comes from the experimental method of measuring the backscattered current in a nonchiral LL, for which the right-moving modes and the left-moving modes are not spatially separated. This backscattered current cannot be measured directly as in a fractional quantum Hall edge system but it is assumed to be the difference between the current in the absence of an impurity (estimated using the ideal conductance of a nanotube $4e^2/h$) and the measured current.

We claim that the combination of these three factors yields a reduced value of the Fano factor consistent with the experimental measurements. Indeed, the reduction in the ideal conductance of the nanotube from $4e^2/h$ to $4g_e e^2/h$ generates large errors in the evaluation of the backscattered current (that use the *noninteracting* conductance $4e^2/h$). Moreover, the zero-frequency noise is also reduced from eI_B to $g_e eI_B$. This yields a reduction in the "true" Fano factor of the system to $g_e e$ instead of e. The explanation proposed in this theoretical paper is compatible and consistent with experimental results of Ref. 7. Our analysis also allows us to propose a direct experimental test to extract the value of g_e from the dependence of the zero-temperature noise on the conductance of the wire.

A quantum wire connected to metallic leads is presented in Fig. 1. The interaction parameter $g(x)$ is space dependent and its value is g in the bulk of the wire and 1 in the leads.

The ideal ac conductance for this system in the absence of impurity scattering and in the absence of decoherence is given by[3,11]

$$G_0(\omega) = \frac{e^2}{h}\left[1 + \gamma \frac{2i\,\sin(\omega/\omega_L)}{e^{-i\omega/\omega_L} - \gamma^2 e^{i\omega/\omega_L}}\right], \qquad (1)$$

where $\gamma = (1-g)/(1+g)$, $\omega_L = v_F/gL$, v_F is the Fermi velocity, and L is the length of the wire. The effects of decoherence

FIG. 1. A quantum wire with an impurity adiabatically coupled to metallic leads. The leads are held at different chemical potentials μ_1 and μ_2.

can at first naively be taken into account by adding an imaginary self-energy to the propagating Green's function or equivalently using the standard field theory procedure of adding an imaginary part to the energy such that $\omega \to \omega + i\delta$, where $\delta \propto 1/\tau$ is the inverse quasiparticle lifetime.

As discussed in the introduction, some physical factors that may contribute to a finite lifetime for the fractionally charged quasiparticles include the curvature of the bands in the nanotube at higher energies. However, in order to estimate exactly the value of this lifetime (which may even depend on momentum and energy), as well as of the conductance of a nanotube in contact with metallic leads in the presence of interactions and decoherence, more extensive calculations are necessary which are beyond the scope of this work and will be addresses in a future publication.

We focus mainly on the dc conductance and take the limit $\omega \to 0$ while keeping δ finite. This yields

$$G_0 = \frac{e^2}{h} \left[1 + \gamma \frac{2 \sinh(\delta/\omega_L)}{e^{\delta/\omega_L} - \gamma^2 e^{-\delta/\omega_L}} \right]. \quad (2)$$

We can study a few limits of Eq. (2). For example, if the tube is much shorter than the coherence length, such that $\delta \ll \omega_L$, the conductance of the wire goes back to the noninteracting value of $G_0 = e^2/h$. In the opposite limit, when the tube is much longer than the coherence length we find a value for the conductance of $G_0 = 2g/(1+g)e^2/h$, which is the conductance of an ideal semi-infinite wire-metal junction. In the intermediate regime, for example, if the coherence length is a few times the length of the wire, the conductance is reduced to $G_0 = g_e e^2/h$, with g_e close to but slightly smaller that 1. We believe this situation is most relevant for the existing experiments. Also, for a nanotube which has four channels of conduction $G_0 = 4g_e e^2/h$.

In the presence of an impurity, a portion of the current will be backscattered, such that $I = I_0 - I_B$, with $I_0 \equiv G_0 V$. In general, at low temperatures and for voltages larger than ω_L, the backscattered current has a nonlinear dependence on voltage,[3,12] consistent with the LL theory. However, for voltages smaller than ω_L, the dependence of the backscattered current on voltage is linear.[3,12] The impurity-induced noise in the current is[13]

$$S = 2g_e e I_B \mathcal{T}(E) \coth\left(\frac{g_e e V}{2 k_B T}\right) + 4 k_B T G_0 \mathcal{T}^2(E), \quad (3)$$

where $\mathcal{T}(E) \equiv \frac{1}{G_0} \frac{dI}{dV}\big|_{eV=E}$ is the energy-dependent transmission of the wire, I is the transmitted current, and the backscattered current is

$$I_B = G_0 V - I = G_0 V - \int_0^{eV} dE' \frac{dI}{edV}\bigg|_{eV=E}. \quad (4)$$

For $g_e = 1$, Eq. (3) describes adequately the noise for a noninteracting system.[14] For an infinite LL ($g_e = g$), Eq. (3) correctly describes the noise if the backscattering is small ($\mathcal{T}(E) \approx 1$), moreover this formula is accurate up to a few percent error even for lower transmission coefficients and up to a bare transmission of the barrier of $\approx 50\%$.[13] Equation (3) also describes accurately the noise of an interacting LL with leads;[3,15] for this system it was proposed that g_e is equal to 1. The arguments we presented above for the ideal conductance of a nonchiral LL connected to metallic leads imply that Eq. (3) with a renormalized of $g_e \approx 0.9$ should be used to describe the noise in a nanotube.

We can now estimate the value of the Fano factor and compare it with the experimentally measured value. The Fano factor is given by the ratio between the noise and the backscattered currents. The noise is measured directly in experiments. However the backscattered current for a nonchiral LL cannot be measured directly but is inferred[7] from the measured transmitted current, I, using

$$I_B^* = 4\frac{e^2}{h} V - \int_0^{eV} dE' \frac{dI}{edV}\bigg|_{eV=E}. \quad (5)$$

Thus, in the limit of small temperature, the experimentally inferred Fano factor is given by

$$F^* = \frac{S}{2e I_B^*} = \frac{g_e I_B \mathcal{T}(E)}{I_B^*} = g_e \mathcal{T}(E) \frac{4g_e \frac{e^2}{h} V - \int_0^{eV} dE' \frac{dI}{edV}\big|_{eV=E}}{4\frac{e^2}{h} V - \int_0^{eV} dE' \frac{dI}{edV}\big|_{eV=E}}. \quad (6)$$

One should note that the real Fano factor, $S/2e I_B = g_e \mathcal{T}(E)$, would be accessible if one could measure directly the backscattered current. In the limit of good transmission, $\mathcal{T}(E) \approx 1$, the real Fano factor would yield the value of the fractional charge g_e. However, given the experimental difficulties to isolate the backscattered current, one estimates instead F^*. It is not hard to see how the value of F^* may easily differ from the noninteracting value 1, even if g_e is very close to 1. For example, assuming that $I = \int_0^{eV} dE/edI/dV|_{eV=E} \equiv \alpha 4g_e e^2/hV$, ($\alpha$ acts as the transmission coefficient for a wire with the ideal conductance $g_e e^2/h$ and is always smaller than 1), in the limit of very good transmission $\mathcal{T}(E) \approx 1$, $(1 - \alpha \ll 1)$ we get

$$F^* \approx g_e^2 \frac{1 - \alpha}{1 - g_e \alpha}. \quad (7)$$

Thus, for $\alpha = 0.9$, and $g_e = 0.9$, $F^* \approx 0.4$, in very good agreement with the experimental observations. Note that this value is reduced further by a factor of $\mathcal{T}(E)$ in the regime where the transmission deviates significantly from unity.

An important observation is that the measured Fano factor F^* depends strongly on the transmission of the wire α (decreasing toward zero with increasing α toward a perfect

transmission). On the other hand the real Fano factor is roughly independent or increases weakly with increasing the differential conductance. Moreover, in the regime where the dependence of the current on voltage is nonlinear, α will also depend on voltage, and the measured Fano factor F^* should also depend on the voltage.

We also propose a direct test of our hypothesis and a direct way of measuring g_e by analyzing the dependence of the zero-temperature noise in units of $2eVdI/dV$ on I/V. Rewriting Eq. (3) as

$$S = 2g_e e(V - I/G_0)dI/dV, \qquad (8)$$

we can see that

$$\widetilde{S} = S/(2eVdI/dV) = g_e[1 - 1/G_0 I/V]. \qquad (9)$$

Thus one expects $\widetilde{S}$ to have a linear dependence on I/V, with a $g_e/G_0 = h/4e^2$ slope but with an x axis intercept at $x_0 = G_0 = g_e 4e^2/h$. The experimental data presented in Ref. 7 depict only S/I as a function of dI/dV, which should have a slightly different behavior but for which we can already observe a small deviation from 1 in the intercept of the fit to the data with the x axis. The data presented in Ref. 7 is however hard to interpret due to both finite temperature effects, as well as nonlinearities in $\mathcal{T}(E)$ (due to the scattering of quasiparticles at the two contacts). More precise data at lower temperature is needed for a detailed quantitative comparison with the theoretical predictions.

In conclusion, we have found that the interplay between decoherence and interactions in nonchiral LL's can explain the recent experimental observation of Ref. 7 that finds a substantial reduction in the Fano factor from the noninteracting value. Indeed, it has been predicted that the Fano factor of a Luttinger liquid connected with metallic leads should be the same as that of a system without interactions. Our analysis has revealed that in the presence of interactions and decoherence the *true* Fano factor of the wire is reduced to a value g_e that is close but not equal to 1.

Furthermore, in an experiment one does not have access to the true Fano factor but to the *inferred* Fano factor, which is computed using a value of the backscattered current that is not measured directly but estimated indirectly from the measured conductance using the noninteracting ideal conductance. We have demonstrated that even small reductions to the *true* Fano factor and to the ideal conductance dramatically lower the inferred Fano factor and can explain the value observed in the experiments presented in Ref. 7.

Last, but not least, we have proposed a direct test for extracting the value of the renormalized fractional-charge parameter g_e from the dependence of the zero-temperature noise (renormalized by the voltage and the differential conductance) on I/V. A more comprehensive study of the Fano factor in the presence of interactions and decoherence, as well as an analysis of the interplay between various sources of decoherence is under way.

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